

# 1 Cell Population Growth Kinetics in 2 the Presence of Stochastic 3 Heterogeneity of Cell Phenotype

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11 **Abstract** Recent studies at individual cell resolution have revealed phenotypic heterogeneity in  
12 nominally clonal tumor cell populations. The heterogeneity affects cell growth behaviors, which can  
13 result in departure from the idealized exponential growth. Here we measured the stochastic time  
14 courses of growth of an ensemble of populations of HL60 leukemia cells in cultures, starting with  
15 distinct initial cell numbers to capture the departure from the exponential growth model in the  
16 initial growth phase. Despite being derived from the same cell clone, we observed significant  
17 variations in the early growth patterns of individual cultures with statistically significant differences  
18 in growth kinetics and the presence of subpopulations with different growth rates that endured for  
19 many generations. Based on the hypothesis of existence of multiple inter-converting  
20 subpopulations, we developed a branching process model that captures the experimental  
21 observations.

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## 23 Introduction

24 Cancer has long been considered a genetic disease caused by oncogenic mutations in somatic  
25 cells that confer a proliferation advantage. According to the clonal evolution theory, accumulation  
26 of random genetic mutations produces cell clones with cancerous cell phenotype. Specifically,  
27 cells with the novel genotype(s) may display increased proliferative fitness and gradually out-  
28 grow the normal cells, break down tissue homeostasis and gain other cancer hallmarks (*Hanahan  
29 and Weinberg, 2011*). In this view, a genetically distinct clone of cells dominates the cancer cell  
30 population and is presumed to be uniform in terms of the phenotype of individual cells within an  
31 isogenic clone. In this traditional paradigm, non-genetic phenotypic variation within one clone is  
32 not taken into account.

33 With the advent of systematic single-cell resolution analysis, however, non-genetic cell hetero-  
34 geneity within clonal (cancer) cell populations is found to be universal (*Pisco and Huang, 2015*). This  
35 feature led to the consideration of the possibility of biologically (qualitatively) distinct (meta)stable  
36 cell subpopulations due to gene expression noise, representing intra-clonal variability of features  
37 beyond the rapid random micro-fluctuations. Hence, transitions between the subpopulations, as  
38 well as heterotypic interactions among them may influence cell growth, migration, drug resistance,  
39 etc. (*Tabassum and Polyak, 2015; Gunnarsson et al., 2020; Durrett et al., 2011*). Thus, an emerging  
40 view is that cancer is more akin to an evolving ecosystem (*Gatenby et al., 2014*) in which cells form

41 distinct subpopulations with persistent characteristic features that determine their mode of inter-  
42 action, directly or indirectly via competition for resources (*Egeblad et al., 2010; Sonnenschein and*  
43 *Soto, 2000*). However, once non-genetic dynamics is considered, cell “ecology” differs fundamentally  
44 from the classic ecological system in macroscopic biology: the subpopulations can reversibly switch  
45 between each other whereas species in an ecological population do not convert between each  
46 other (*Clark, 1991*). This affords cancer cell populations a remarkable heterogeneity, plasticity and  
47 evolvability, which may play important roles in their growth and in the development of resistance to  
48 treatment (*Meacham and Morrison, 2013*).

49 Many new questions arise following the hypothesis that phenotypic heterogeneity and transi-  
50 tions between phenotypes within one genetic clone are important factors in cancer. Can tumors  
51 arise, as theoretical considerations indicate, because of a state conversion (within one clone) to  
52 a phenotype capable of faster, more autonomous growth as opposed to acquisition of a new  
53 genetic mutation that confers such a selectable phenotype (*Zhou et al., 2014a; Angelini et al., 2022;*  
54 *Howard et al., 2018; Sahoo et al., 2021; Pisco and Huang, 2015; Zhou et al., 2014b; Kochanowski*  
55 *et al., 2021*)? Is the macroscopic, apparently sudden outgrowth of a tumor driven by a new fastest-  
56 growing clone (or subpopulation) taking off exponentially, or due to the cell population reaching a  
57 critical mass that permits positive feedback between its subpopulations that stimulates outgrowth,  
58 akin to a collectively autocatalytic set (*Hordijk et al., 2018*)? Should therapy target the fastest  
59 growing subpopulations, or target the interactions and interconversions of cancer cells?

60 At the core of these deliberations is the fundamental question on the mode of tumor cell  
61 population growth that now must consider the influence of inherent phenotypic heterogeneity  
62 of cells and the non-genetic (hence potentially reversible) inter-conversion of cells between the  
63 phenotypes that manifest various growth behaviors and the interplay between these two modalities.

64 Traditionally tumor growth has been described as following an exponential growth law, moti-  
65 vated by the notion of uniform cell division rate for each cell, i.e. a first order growth kinetics  
66 (*Mackillop, 1990*). But departure from the exponential model has long been noted. To better fit  
67 experimental data, two major modifications have been developed, namely the Gompertz model  
68 and the West law model (*Yorke et al., 1993*). While no one specific model can adequately describe  
69 any one tumor, each model highlights certain aspects of macroscopic tumor kinetics, mainly the  
70 maximum size and the change in growth rate at different stages. These models however are not  
71 specifically motivated by cellular heterogeneity. Assuming non-genetic heterogeneity with transi-  
72 tions between the cell states, the population behavior is influenced by many intrinsic and extrinsic  
73 factors that are both variable and unpredictable at the single-cell level. Thus, tumor growth cannot  
74 be adequately captured by a deterministic model, but a stochastic cell and population level kinetic  
75 model is more realistic.

76 Using stochastic processes in modeling cell growth via clonal expansion has a long history  
77 (*Zheng, 1999*). An early work is the *Luria and Delbrück (1943)* model, which assumes cells grow  
78 deterministically, with wildtype cells mutating and becoming (due to rare and quasi-irreversible  
79 mutations) cells with a different phenotype randomly. Since then, there have been many further  
80 developments that incorporate stochastic elements into the model, such as those proposed by  
81 *Lea and Coulson (1949), Koch (1982), Luebeck and Moolgavkar (2002), and Dewanji et al. (2005)*.  
82 We can find various stochastic processes: Poisson processes (*Bartoszynski et al., 1981*), Markov  
83 chains (*Gupta et al., 2011*), and branching processes (*Jiang et al., 2017*), or even random sums  
84 of birth-death processes (*Dewanji et al., 2005*), all playing key roles in the mathematical theories  
85 of cellular clonal growth and evolution. These models have been applied to clinical data on lung  
86 cancer (*Newton et al., 2012*), breast cancer (*Speer et al., 1984*), and treatment of cancer (*Spina*  
87 *et al., 2014*).

88 At single-cell resolution, another cause for departure from exponential growth is the presence  
89 of positive (growth promoting) cell-cell interactions (Allee effect) in the early phase of population  
90 growth, such that cell density plays a role in stimulating division, giving rise to the critical mass  
91 dynamics (*Johnson et al., 2019; Korolev et al., 2014*).

92 To understand the intrinsic tumor growth behavior (change of tumor volume over time) it is  
93 therefore essential to study tumor cell populations in culture which affords detailed quantitative  
94 analysis of cell numbers over time, unaffected by the tumor microenvironment, and to measure  
95 departure from exponential growth. This paper focuses on stochastic growth of clonal but pheno-  
96 typically heterogeneous HL60 leukemia cells with near single-cell sensitivities in the early phase  
97 of growth, that is, in sparse cultures. We and others have in the past years noted that at the  
98 level of single cells, each cell behaves akin to an individual, differently from another, which can be  
99 explained by the slow correlated transcriptome-wide fluctuations of gene expression (*Chang et al.,*  
100 *2008; Li et al., 2016*). Given the phenotypic heterogeneity and anticipated functional consequences,  
101 grouping of cells is necessary. Such classification would require molecular cell markers for said  
102 functional implication, but such markers are often difficult to determine a priori. Here, since most  
103 pertinent to cancer biology, we directly use a functional marker that is of central relevance for  
104 cancer: cell division, which maps into cell population growth potential — in brief “cell growth”.

105 Therefore, we monitored longitudinally the growth of cancer cell populations seeded at very  
106 small numbers of cells (1, 4, or 10 cells) in statistical ensembles of microcultures (wells on a plate of  
107 wells). We found evidence that clonal HL60 leukemia cell populations contain subpopulations that  
108 exhibit diverse growth patterns. Based on statistical analysis, we propose the existence of three  
109 distinctive cell phenotypic states with respect to cell growth. We show that a branching process  
110 model captures the population growth kinetics of a population with distinct cell subpopulations. Our  
111 results suggest that the initial phase cell growth (“take-off” of a cell culture) in the HL60 leukemic cells  
112 is predominantly driven by the fast-growing cell subpopulation. Reseeding experiments revealed  
113 that the fast-growing subpopulation could maintain its growth rate over several cell generations,  
114 even after the placement in a new environment. Our observations underscore the need to not only  
115 target the fast-growing cells but also the transition to them from the other cell subpopulations.

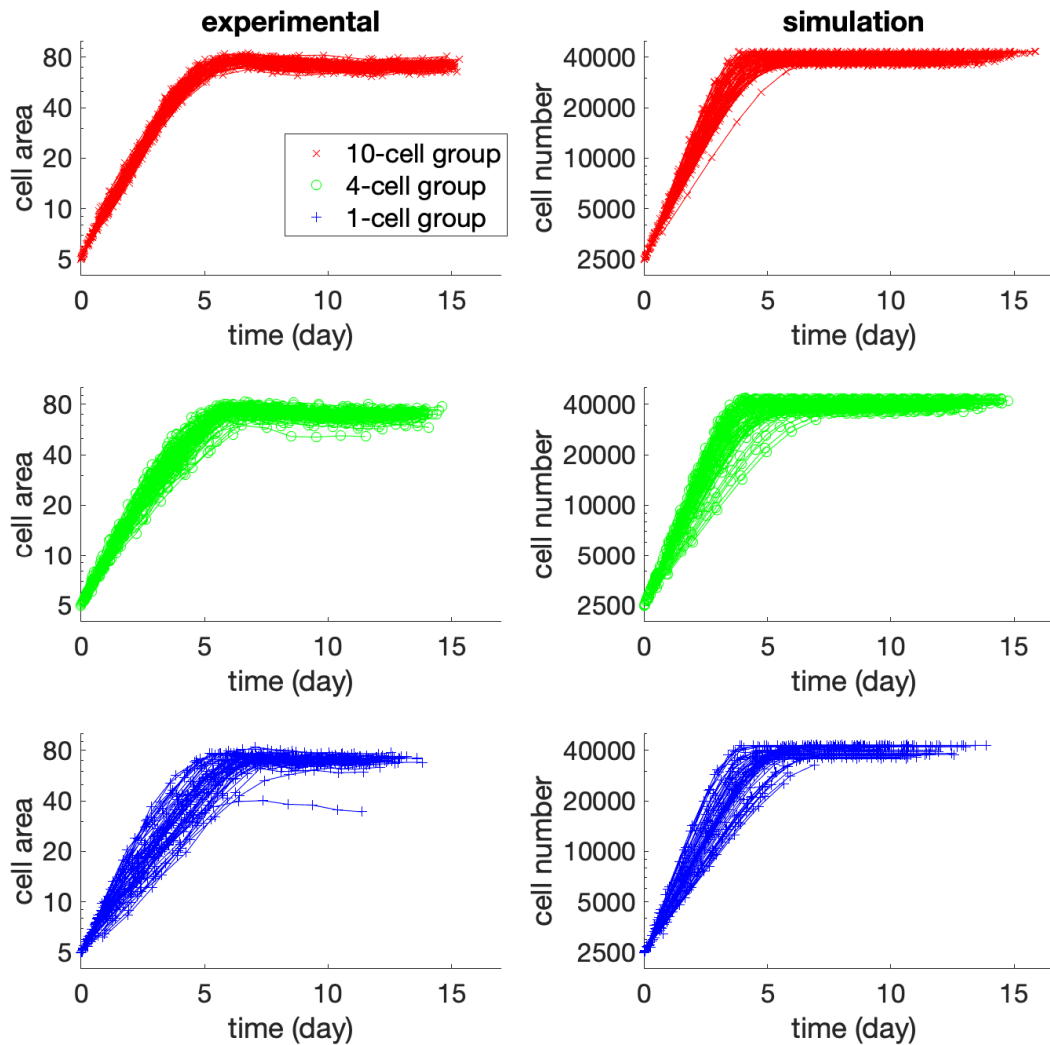
## 116 Results

### 117 Experiment of the cell population growth from distinct initial cell numbers.

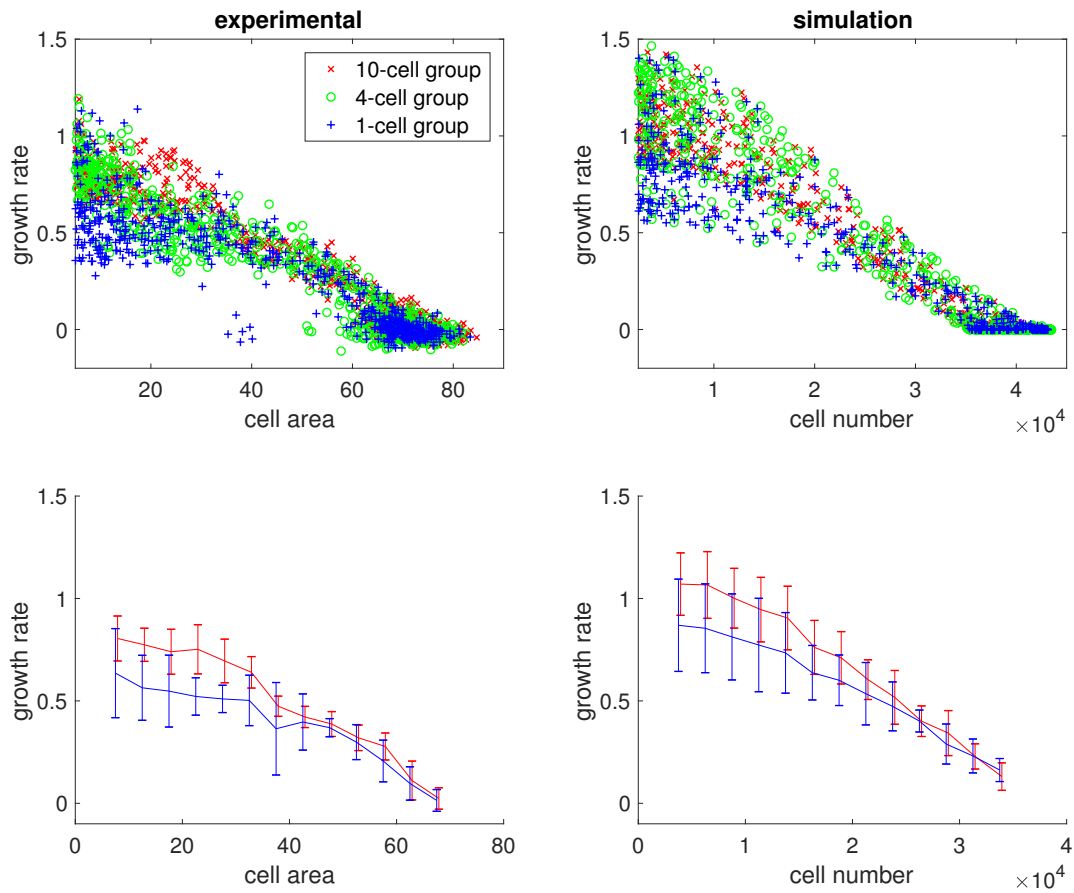
118 To expose the variability of growth kinetics as a function of initial cell density  $N_0$  (“initial seed  
119 number”), HL60 cells were sorted into wells of a 384-well plate (0.084 cm<sup>2</sup> area) to obtain “statistical  
120 ensembles” of replicate microcultures (wells) of the same condition, distinct only by  $N_0$ . Based on  
121 prior titration experiments to determine ranges of interest for  $N_0$  and statistical power, for this  
122 experiment we plated 80 wells with  $N_0 = 10$  cells ( $N_0 = 10$ -cell group), 80 wells with  $N_0 = 4$  cells  
123 ( $N_0 = 4$ -cell group), and 80 wells with  $N_0 = 1$  cell ( $N_0 = 1$ -cell group). Cells were grown in the same  
124 conditions for 23 days (for details of cell culture and sorting, see the Methods section). Digital  
125 images were taken every 24 hours for each well from Day 4 on, and the area occupied by cells in  
126 each well was determined using computational image analysis. We had previously determined that  
127 one area unit equals approximately 500 cells. This is consistent and readily measurable because the  
128 relatively rigid and uniformly spherical HL60 cells grow as a non-adherent “packed” monolayer at  
129 the bottom of the well. Note that we are interested in the initial exponential growth (and departure  
130 from it) and not in the latter phases when the culture becomes saturated as has been the historical  
131 focus of analysis (see Introduction).

132 Wells that have reached at least 5 area units were considered for the characterization of early  
133 phase (before plateau) growth kinetics by plotting the areas in logarithmic scale as a function of  
134 time (Fig. 1). All the  $N_0 = 10$ -cell wells required 3.6-4.6 days to grow from 5 area units to 50 area  
135 units (mean=4.05, standard deviation=0.23). For the  $N_0 = 1$ -cell wells, we observed a diversity of  
136 behaviors. While some of the cultures only took 3.5-5 days to grow from 5 area units to 50 area  
137 units, others needed 6-7.2 days (mean=5.02, standard deviation=0.75). The  $N_0 = 4$ -cell wells had a  
138 mean=4.50 days and standard deviation=0.44 to reach that same population size.

139 To examine the exponential growth model, in Fig. 2 (left panel), we plotted the per capita growth  
140 rate versus cell population size, where each point represents a well (population) at a time point.



**Figure 1.** Growth curves of the experiment (left) and simulation (right), starting from the time of reaching 5 area units (experiment) or having 2500 cells (simulation), with a logarithm scale for the  $y$ -axis. The time required for reaching 5 area units was determined by exponential extrapolation, as reliable imaging started at  $> 5$  area units. The  $x$ -axis is the time from reaching 5 area units (experiment) or 2500 cells (simulation). Red, green, or blue curves correspond to 10, 4, or 1 initial cell(s). Although starting from the same population level, patterns are different for distinct initial cell numbers. The  $N_0 = 1$ -cell group has higher diversity.



**Figure 2.** Per capita growth rate (averaged within one day) vs. cell population for the experiment (left) and simulation (right). Each point represents one well in one day. Red, green, or blue points correspond to 10, 4, or 1 initial cell(s).

141 As expected, as the population became crowded, the growth rate decreased toward zero. But in  
142 the earlier phase, many populations in the  $N_0 = 1$ -cell group had a lower per capita growth rate  
143 than those in the  $N_0 = 10$ -cell group, even at the same population size – thus departing from the  
144 expected behavior of exponential growth. The weighted Welch's  $t$ -test showed that the difference in  
145 these growth rates was significant (see the Methods section).

146 While qualitative differences in the behaviors of cultures with different initial seeding cell  
147 numbers  $N_0$  can be expected for biological reasons (see below), in the elementary exponential  
148 growth model, the difference of growth rate should disappear when populations with distinct  
149 seeding numbers are aligned for the same population size that they have reached as in Fig. 2. A  
150 simple possibility is that the deviations of expected growth rates emanate from difference in cell-  
151 intrinsic properties. Some cells grew faster, with a per capita growth rate of  $0.6 \sim 0.9$  (all  $N_0 = 10$ -cell  
152 wells and some  $N_0 = 1$ -cell wells), while some cells grew slower, with a per capita growth rate of  
153  $0.3 \sim 0.5$  (some of the  $N_0 = 1$ -cell wells). In other words, there is intrinsic heterogeneity in the cell  
154 population that is not “averaged out” in the culture with low  $N_0$ , and the sampling process exposes  
155 these differences between the cells that appear to be relatively stable.

156 To illustrate the inherent diversity of initial growth rates, in Fig. 3 (left panel), we display the daily  
157 cell-occupied areas plotted on a linear scale starting from Day 4. All wells with seed of  $N_0 = 10$  or  
158  $N_0 = 4$  cells grew exponentially. Among the  $N_0 = 1$ -cell wells, 14 populations died out. Four wells  
159 in the  $N_0 = 1$ -cell group had more than 10 cells on Day 8 but never grew exponentially, and had  
160 fewer than 1000 cells after 15 days (on Day 23). For these non-growing or slow-growing  $N_0 = 1$ -cell  
161 wells, the per capita growth rate was  $0 \sim 0.2$ . In comparison, all the  $N_0 = 10$ -cell wells needed at  
162 most 15 days to reach the carrying capacity (around 80 area units, or 40000 cells). See Table 1 for a  
163 summary of the  $N_0 = 1$ -cell group's growth patterns. This behavior is not idiosyncratic to the culture  
164 system because they recapitulate a pilot experiment performed in the larger scale format of 96-well  
165 plates (not shown).

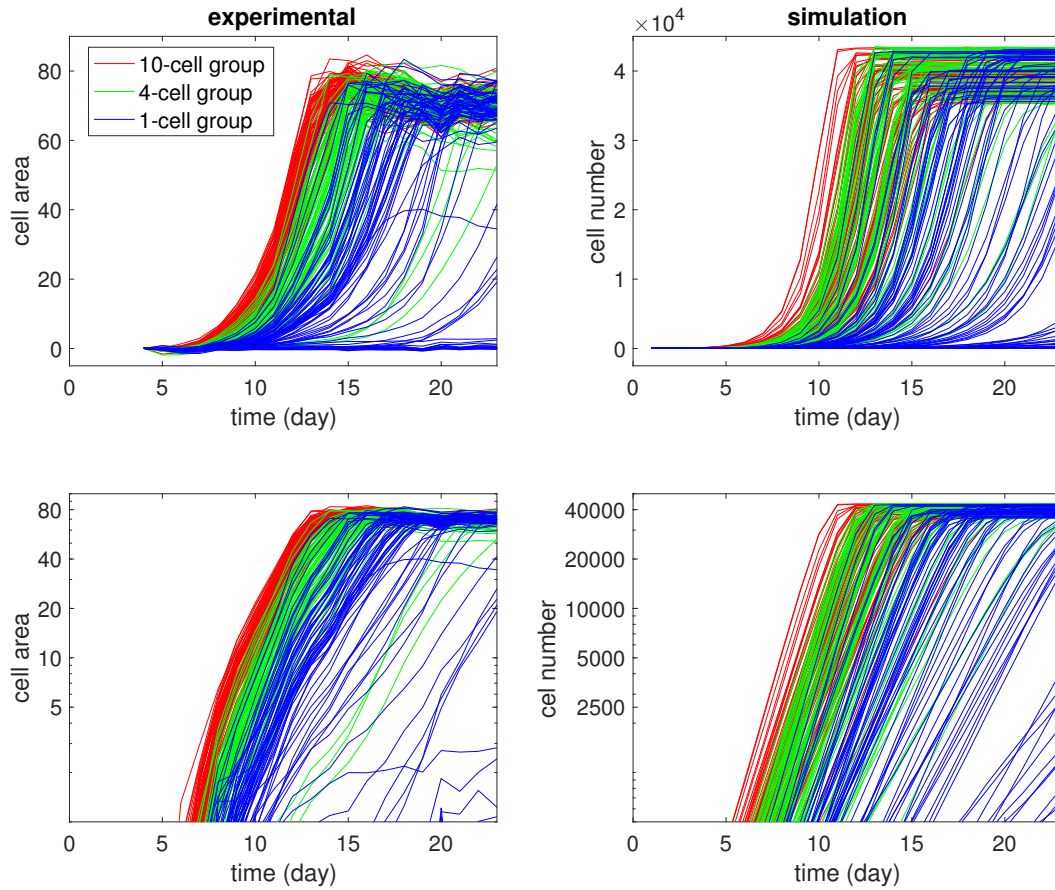
166 From the above experimental observations, we asserted that there might be at least three  
167 stable cell growth phenotypes in a population: a fast type, whose growth rate was  $0.6 \sim 0.9$ /day for  
168 non-crowded conditions; a moderate type, whose growth rate was  $0.3 \sim 0.5$ /day for non-crowded  
169 conditions; and a slow type, whose growth rate was  $0 \sim 0.2$ /day for the non-crowded population.

170 The graphs of Fig. 3 also revealed other phenomena of growth kinetics: (1) Most  $N_0 = 4$ -cell  
171 wells plateaued by Day 14 to Day 17, but some lagged significantly behind. (2) Similarly, four wells  
172 in the  $N_0 = 1$ -cell group exhibited longer lag-times before the exponential growth phase, and never  
173 reached half-maximal cell numbers by Day 23. These outliers reveal intrinsic variability and were  
174 taken into account in the parameter scanning (see the Methods section).

### 175 **Reseeding experiments revealing the enduring intrinsic growth patterns.**

176 When a well in the  $N_0 = 1$ -cell group had grown to 10 cells, population behavior was still different  
177 from those in the  $N_0 = 10$ -cell group at the outset. In view of the spate of recent results revealing  
178 phenotypic heterogeneity, we hypothesized that the difference was cell-intrinsic as opposed to  
179 being a consequence of the environment (e.g., culture medium in  $N_0 = 1$  vs  $N_0 = 10$ -cell wells).

180 To test our hypothesis and exclude differences in the culture environment as determinants of  
181 growth behavior, we reseeded the cells that exhibited the different growth rates in fresh cultures.  
182 We started with a number of  $N_0 = 1$ -cell wells. After a period of almost 3 weeks, again some wells  
183 showed rapid proliferation, with cells covering the well, while others were half full and yet others  
184 wells were almost empty. We collected cells from the full and half-full wells and reseeded them into  
185 32 wells each (at about  $N_0 = 78$  cells per well). These 64 wells were monitored for another 20 days.  
186 We found that most wells reseeded from the full well took around 11 days to reach the population  
187 size of a half-full well, while most wells reseeded from the half-full well required around  $16 \sim 20$   
188 days to reach the same half full well population size. Five wells reseeded from the half-full wells  
189 were far from even reaching half full well population size by Day 20 (see Table 2). Permutation test  
190 showed that this difference in growth rate was significant (see the Methods section).



**Figure 3.** Growth curves of the experiments with different initial cell numbers  $N_0$  (left) and growth curves of corresponding simulation (right). Each curve describes the change in the cell population (measured by area or number) over a well along time. Red, green, or blue curves correspond to  $N_0 = 10, 4,$  or 1 initial cell(s).

**Table 1.** The population of some wells in the  $N_0 = 1$ -cell group in the growth experiment with different initial cell numbers, where  $\sim$  meant approximate cell number. These wells illustrated different growth patterns from those wells starting with  $N_0 = 10$  or  $N_0 = 4$  cells. Such differences implied that cells from wells with different initial cell numbers were essentially different.

Growth pattern	Well label	Day 1	Day 8	Day 14	Day 19	Day 23
No growth, extinction	162,167,170,176,	1	<10	<10	~0	Empty
	177,179,182,183,					
	186,201,234,236,					
	239,240					
Slow growth, no exponential growth	165	1	89	~300	~350	~500
	166	1	36	~110	~120	~150
	178	1	43	~140	~170	~200
	211	1	16	~90	~200	~400
Delayed exponential growth	163	1	12	~130	~300	~5000
	181	1	44	~270	~550	~5500
	193	1	25	~200	~800	~9000
	204	1	21	~100	~600	~6000
Normal exponential growth	200 and many others	1	~130	~20000	~40000 (full)	~40000 (full)

**Table 2.** The distribution of time needed for each well to reach the “half area” population size in the reseeding experiment. We reseeded equal numbers of cells that grew faster (from a full well) and cells that grew slower (from a half-full well), and cultivated them under the same new fresh medium environment to compare their intrinsic growth rates. The results showed that faster growing cells, even reseeded, still grew faster.

Time (days) to reach one half area	11	12	13	14	15	16–20	>20
Faster wells	26	2	1	2	1	0	0
Slower wells	0	0	0	1	1	25	5

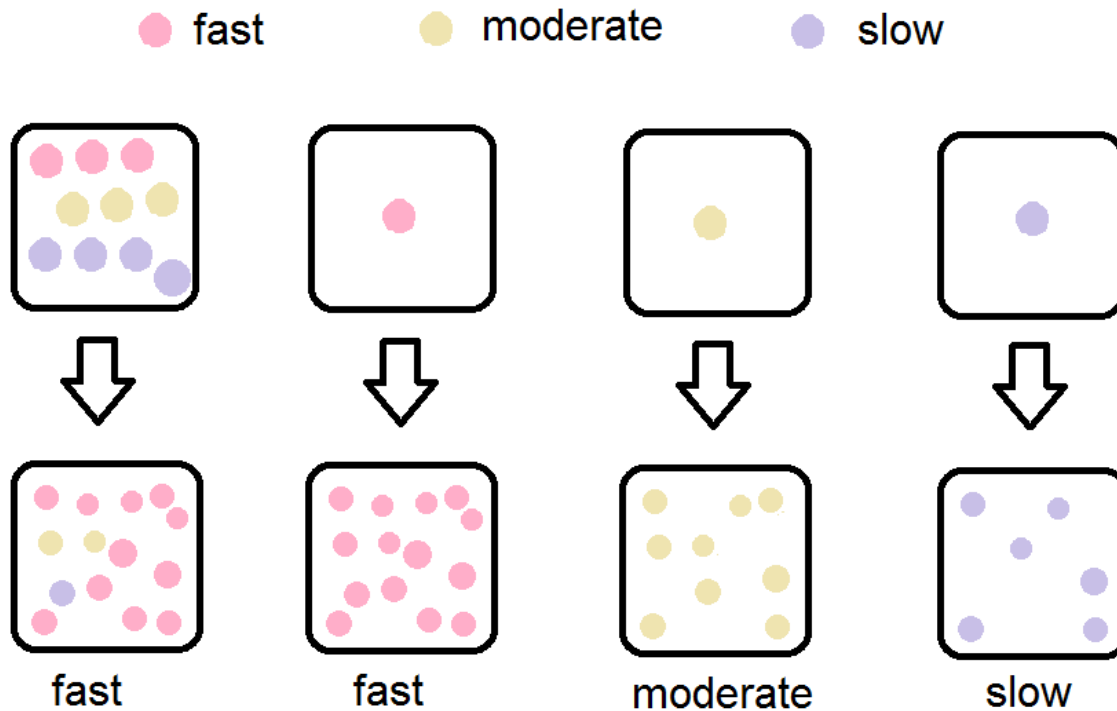
191 This reseeding experiment shows that the difference in growth rate was maintained over multiple  
 192 generations, even after slowing down in the plateau phase (full well) and was maintained when  
 193 restarting a microculture at low density in fresh medium devoid of secreted cell products. Therefore,  
 194 it is plausible that there exists endogenous heterogeneity of growth phenotypes in the clonal HL60  
 195 cell line and that these distinct growth phenotypes are stable for at least 15 ~ 20 cell generations.

### 196 Quantitative analysis of experimental results.

197 In the experiments with different initial cell numbers  $N_0$ , we observed at least three patterns with  
 198 different growth rates, and the reseeding showed that these growth patterns were endogenous to  
 199 the cells. Therefore, we propose that each growth pattern discussed above corresponded to a cell  
 200 phenotype that dominated the population: fast, moderate, and slow.

201 In the initial seeding of cells that varies  $N_0$ , the cells were randomly chosen (by FACS); thus, their  
 202 intrinsic growth phenotypes were randomly distributed. During growth, the population of a well  
 203 would be dominated by the fastest type that existed in the seeding cells, thus qualitatively, we have  
 204 following scenarios: (1) A well in the  $N_0 = 10$ -cell group almost certainly had at least one initial cell





**Figure 4.** Schematic illustration of the qualitative argument: Three cell types and growth patterns (three colors) with different seeding numbers. One  $N_0 = 10$ -cell well will have at least one fast type cell with high probability, which will dominate the population. One  $N_0 = 1$ -cell well can only have one cell type, thus in the microculture ensemble of replicate wells, three possible growth patterns for wells can be observed.

205 of fast type, and the population would be dominated by fast type cells. Different wells had almost  
206 the same growth rate, reaching saturation at almost the same time. (2) For an  $N_0 = 1$ -cell well, if  
207 the only initial cell is of the fast type, then the population has only the fast type, and the growth  
208 pattern will be close to that of  $N_0 = 10$ -cell wells. If the only initial cell is of the moderate type, then  
209 the population could still grow exponentially, but with a slower growth rate. This explains why after  
210 reaching 5 area units, many but not all  $N_0 = 1$ -cell wells were slower than  $N_0 = 10$ -cell wells. (3)  
211 Moreover, in such an  $N_0 = 1$ -cell well with a moderate type initial cell, the cell might not divide quite  
212 often during the first few days due to randomness of entering the cell cycle. This would lead to a  
213 considerable delay in entering the exponential growth phase. (4) By contrast, for an  $N_0 = 1$ -cell well  
214 with a slow type initial cell, the growth rate could be too small, and the population might die out  
215 or survive without ever entering the exponential growth phase in duration of the experiment. (5)  
216 Most  $N_0 = 4$ -cell wells had at least one fast type initial cell, and the growth pattern was the same as  
217  $N_0 = 10$ -cell wells. A few  $N_0 = 4$ -cell wells only had moderate and slow cells, and thus had slower  
218 growth patterns.

219 The above verbal argument is shown in Fig. 4 and entails mathematical modeling with the ap-  
220 propriate parameters that relate the relative frequency of these cell types in the original population,  
221 their associated growth and transition rates to examine whether it explains the data.

### 222 **Branching process model.**

223 To construct a quantitative dynamical model to recapitulate the growth dynamics differences from  
224 cell populations with distinct initial seed cell numbers  $N_0$ , and three intrinsic types of proliferation  
225 behaviors, we used a multi-type discrete-time branching process.

226 The traditional method of population dynamics based on ordinary differential equation (ODE),  
227 which is deterministic and has continuous variables, is not suited when the cell population is small

228 as is the case for the earliest stage of proliferation from a few cells being studied in our experiments.  
229 Deterministic models are also unfit because with such small populations and measurements at  
230 single-cell resolution, stochasticity in cell activity does not average out. The nuanced differences  
231 between individual cells cannot be captured by a different deterministic mechanism of each  
232 individual cell, and the only information available is the initial cell number. Thus, the unobservable  
233 nuances between cells are taken care of by a stochastic model.

234 Given the small populations, our model should be purely stochastic, without deterministic  
235 growth. The focus is the concrete population size of a finite number (three) of types, thus Poisson  
236 processes are not suitable. Markov chains can partially describe the proportions under some  
237 conditions, but population sizes are known, not just their ratios, therefore Markov chains are not  
238 necessary. Branching processes can describe the population size of multiple types with symmetric  
239 and asymmetric division, transition, and death (*Jiang et al., 2017*). Also, the parameters can be  
240 temporally and spatially inhomogeneous, which is convenient. Therefore, we utilized branching  
241 processes in our model.

242 In the branching process, each cell during each time interval independently and randomly  
243 chooses a behavior: division, death, or stagnation in the quiescent state, whose rates depend  
244 on the cell growth type. Denoting the growth rate and death rate of the fast type by  $g_F$  and  $d_F$   
245 respectively, and the population size of fast type cells on Day  $n$  by  $F(n)$ , the population at Day  $n + 1$   
246 is:

$$F(n + 1) = \sum_{i=1}^{F(n)} A_i,$$

247 where  $A_i$  for different  $i$  are independent.  $A_i$  represents the descendants of a fast type cell  $i$  after  
248 one day. It equals 2 with probability  $g_F$ , 0 with probability  $d_F$ , and 1 with probability  $1 - g_F - d_F$ .  
249 Therefore, given  $F(n)$ , the distribution of  $F(n + 1)$  is:

$$\mathbb{P}[F(n + 1) = N] = \sum_{2a+b=N} \frac{F(n)!}{a!b![F(n) - a - b]!} g_F^a d_F^{[F(n)-a-b]} (1 - g_F - d_F)^b,$$

250 where the summation is taken for all non-negative integer pairs  $(a, b)$  with  $2a + b = N$ . Moderate  
251 and slow types evolve similarly, with their corresponding growth rates  $g_M, g_S$ , and death rates  $d_M, d_S$ .

252 As shown in Fig. 2, the growth rates  $g_F, g_M$ , and  $g_S$  should be decreasing functions of the total  
253 population. In our model, we adopted a quadratic function.

254 We performed a parameter scan to show that our model could reproduce experimental phe-  
255 nomena for a wide range of model parameters (see details in Table 3).

256 The simulation results are shown on the right panels of Figs. 1–3, in comparison with the  
257 experimental data in the left. Our model qualitatively captured the growth patterns of groups with  
258 different initial seeding cell numbers. For example, in Fig. 2, when wells were less than half full (cell  
259 number  $< 20000$ ), most wells in the  $N_0 = 10$ -cell group grew faster than the  $N_0 = 1$ -cell group even  
260 when they had the same cell number. In Fig. 3, all wells in the  $N_0 = 10$ -cell group in our model grew  
261 quickly until saturation. Similar to the experiment, some wells in the  $N_0 = 1$ -cell group in our model  
262 never grew, while some began to take off very late.

263 In our model, the high extinction rate in the  $N_0 = 1$ -cell group (14/80) was explained as “bad luck”  
264 at the early stage, since birth rate and death rate were close, and a cell could easily die without  
265 division. Another possible explanation for such a difference in growth rates was that the population  
266 would be 10 small colonies when starting from 10 initial cells, while starting from 1 initial cell, the  
267 population would be 1 large colony. With the same area, 10 small colonies should have a larger total  
268 perimeter, thus larger growth space and larger growth rate than that of 1 large colony. However, we  
269 carefully checked the photos, and found that almost all wells produced 1 large colony with nearly  
270 the same shape, and there was no significant relationship between colony perimeter and growth  
271 rate.

**Table 3.** Performance of our model with different parameters. Here we adjusted the parameters of our model in a wide range and observed whether the model could still reproduce four important “features” in the experiment. This parameter scan showed that our model is robust under perturbations on parameters. Here  $p_F, p_M, p_S$  are the probabilities that an initial cell is of fast, moderate, or slow type;  $d$  is the death rate;  $g_0$  is the growth factor;  $r$  is the range of the random modifier. See the Methods section for explanations of these parameters. Feature 1, all wells in the  $N_0 = 10$ -cell group were saturated; Feature 2, presence of late-growing wells in the  $N_0 = 1$ -cell group; Feature 3, presence of non-growing wells in the  $N_0 = 1$ -cell group; Feature 4, different growth rates at the same population size between the  $N_0 = 10$ -cell group and the  $N_0 = 1$ -cell group.

Parameters						Appearance of experimental phenomena			
$p_F$	$p_M$	$p_S$	$d$	$g_0$	$r$	Feature 1	Feature 2	Feature 3	Feature 4
0.4	0.4	0.2	0.01	0.5	0.1	Yes	Yes	Yes	Yes
0.4	0.4	0.2	0	0.5	0.1	Yes	Yes	Yes	Yes
0.4	0.4	0.2	0.05	0.5	0.1	Yes	Yes	Yes	Yes
0.4	0.4	0.2	0.1	0.5	0.1	No	Yes	Yes	No
0.4	0.4	0.2	0.01	0.45	0.1	Yes	Yes	Yes	Yes
0.4	0.4	0.2	0.01	0.6	0.1	Yes	Yes	Yes	Yes
0.4	0.4	0.2	0.01	0.4	0.1	Yes	Yes	Yes	No
0.4	0.4	0.2	0.01	0.5	0.05	Yes	Yes	Yes	Yes
0.4	0.4	0.2	0.01	0.5	0	Yes	Yes	Yes	Yes
0.4	0.4	0.2	0.01	0.5	0.15	Yes	Yes	Yes	No
0.4	0.4	0.2	0.01	0.5	0.2	No	Yes	Yes	No
0.3	0.5	0.2	0.01	0.5	0.1	Yes	Yes	Yes	Yes
0.5	0.3	0.2	0.01	0.5	0.1	Yes	Yes	Yes	Yes
0.4	0.5	0.1	0.01	0.5	0.1	Yes	Yes	Yes	Yes
0.4	0.3	0.3	0.01	0.5	0.1	Yes	Yes	Yes	Yes
0.5	0.4	0.1	0.01	0.5	0.1	Yes	Yes	Yes	Yes
0.3	0.4	0.3	0.01	0.5	0.1	Yes	Yes	Yes	Yes
0.1	0.1	0.8	0.01	0.5	0.1	No	Yes	Yes	No
0.5	0.5	0	0.01	0.5	0.1	Yes	Yes	No	Yes
0	0.5	0.5	0.01	0.5	0.1	No	Yes	Yes	Yes
0.5	0	0.5	0.01	0.5	0.1	Yes	No	Yes	No
1	0	0	0.01	0.5	0.1	Yes	No	No	No

## 272 Discussion

273 As many recent single-cell level data have shown, a tumor can contain multiple distinct subpopulations engaging in interconversions and interactions among them that can influence cancer cell proliferation, death, migration, and other features that contribute to malignancy (Pisco and Huang, 2015; Zhou et al., 2014a; Angelini et al., 2022; Howard et al., 2018; Sahoo et al., 2021; Zhou et al., 2014b; Johnson et al., 2019; Korolev et al., 2014; Chapman et al., 2014; Niu et al., 2015; Chen et al., 2016). Presence of these two intra-population behaviors can be manifest as departure from the elementary model of exponential growth (Skehan and Friedman, 1984) (in the early phase of population growth, far away from carrying capacity of the culture environment which is trivially non-exponential). The exponential growth model assumes uniformity of cell division rates across all cells (hence a population doubling rate that is proportional to a given population size  $N(t)$ ) and the absence of cell-cell interactions that affect cell division and death rates. Investigating the “non-genetic heterogeneity” hypothesis of cancer cells quantitatively is therefore paramount for understanding cancer biology but also for elementary principles of cell population growth.

286 As an example, here we showed that clonal cell populations of the leukemia HL60 cell line are heterogeneous with regard to growth behaviors of individual cells that can be summarized in subpopulations characterized by a distinct intrinsic growth rates which were revealed by analysis of

289 the early population growth starting with microcultures seeded with varying (low) cell number  $N_0$ .  
290 Since we have noted only very weak effect of cell-cell interactions on cell growth behaviors (Allee  
291 effect) in this cell line (as opposed to another cell tumor cell line in which we found that departure  
292 from exponential growth could be explained by the Allee effect (*Johnson et al., 2019*)), we focused  
293 on the very presence among HL60 cells of subpopulations with distinct proliferative capacity as a  
294 mechanism for the departure of the early population growth curve from exponential growth.

295 The reseeding experiment demonstrated that the characteristic growth behaviors of subpopu-  
296 lations could be inherited across cell generations and after moving to a new environment (fresh  
297 culture), consistent with long-enduring endogenous properties of the cells. This result might be  
298 explained by cells occupying distinct stable cell states (in a multi-stable system). Thus, we introduced  
299 multiple cell types with different growth rates in our stochastic model. Specifically, in a branching  
300 process model, we assumed the existence of three types: fast, moderate, and slow cells. The model  
301 we built could replicate the key features in the experimental data, such as different growth rates  
302 at the same population size between the  $N_0 = 10$ -cell group and the  $N_0 = 1$ -cell group, and the  
303 presence of late-growing and non-growing wells in the  $N_0 = 1$ -cell group.

304 While we were able to fit the observed behaviors in which the growth rate depended not  
305 only on  $N(t)$  but also on  $N_0$ , the existence of the three or even more cell types still needs to be  
306 verified experimentally. For instance, statistical cluster analysis of transcriptomes of individual  
307 cells by single-cell RNA-seq (*Bhartiya et al., 2021*) over the population may identify the presence of  
308 transcriptomically distinct subpopulations that could be isolated (e.g., after association with cell  
309 surface markers) and evaluated separately for their growth behaviors.

310 The central assumption of coexistence of multiple subpopulations in the cell line stock must  
311 be accompanied by the second assumption that there are transitions between these distinct cell  
312 populations. For otherwise, in the stock population the fastest growing cell would eventually  
313 outgrow the slow growing cells. Furthermore, one has to assume a steady-state in which the  
314 population of slow growing cells are continuously replenished from the population of fast-growing  
315 cells. Finally, we must assume that the steady-state proportions of the subpopulations are such  
316 that at low seeding wells with  $N_0 = 1$  cells, there is a sizable probability that a microculture receives  
317 cells from each of the (three) presumed subtypes of cells. The number of wells in the ensemble  
318 of replicate microcultures for each  $N_0$ - condition has been sufficiently large for us to make the  
319 observations and inform the model, but a larger ensemble would be required to determine with  
320 satisfactory accuracy the relative proportions of the cell types in the parental stock population.

321 Transitions might also have been happening during our experiment. For example, those late  
322 growing wells in the  $N_0 = 1$ -cell group could be explained by such a transition: Initially, only slow  
323 type cells were present, but once one of these slow growing cells switched to the moderate type, an  
324 exponential growth ensued at the same rate that is intrinsic to that of moderate cells.

325 If there are transitions, what is the transition rate? Our reseeding experiments are compatible  
326 with a relatively slow rate for interconversion of growth behaviors in that the same growth type  
327 was maintained across 30 generations. An alternative to the principle of transition at a constant  
328 intrinsic to each of the types of cells may be that transition is extrinsically determined. Specifically,  
329 the seeding in the “lone” condition of  $N_0 = 1$  may *induce* a dormant state, that is a transition to a  
330 slower growth mode that is then maintained, on average over 30+ generations, with occasional  
331 return to the faster types that account for the delayed exponential growth.

332 This model however would bring back the notion of “environment awareness”, or the principle  
333 of a “critical density” for growth implemented by cell-cell interaction (Allee effect) which we had  
334 deliberately not considered (see above) since it was not necessary. We do not exclude this possibility  
335 which could be experimentally tested as follows: Cultivate  $N_0 = 1$ -cell wells for 20 days when the  
336 delayed exponential growth has happened in some wells, but then use the cells of those wells with  
337 fast-growing population (which should contain of the fast type) to restart the experiment, seeded  
338 at  $N_0 = 10, 4, 1$  cells. If wells with different seeding numbers exhibit the same growth rates, then  
339 the growth difference in the original experiment is solely due to preexisting (slow interconverting)

340 cell phenotypes. If now the  $N_0 = 1$ -cell wells resumes the typical slow growth, this would indicate a  
341 density induced transition to the slow growth type.

342 In the spirit of Occam's razor, and given the technical difficulty in separate experiments to  
343 demonstrate cell-cell interactions in HL60 cells, we were able to model the observed behaviors  
344 with the simplest assumption of cell-autonomous properties, including existence of multiple states  
345 (growth behaviors) and slow transitions between them but without cell density dependence or  
346 interactions.

347 Taken together, we showed that one manifestation of the burgeoning awareness of ubiquitous  
348 cell phenotype heterogeneity in an isogenic cell population is the presence of distinct intrinsic types  
349 of cells that slowly interconvert among them, resulting in a stationary population composition.  
350 The differing growth rates of the subtypes and their stable proportions may be an elementary  
351 characteristic of a given population that by itself can account for the departure of early population  
352 growth kinetics from the basic exponential growth model.

## 353 **Methods**

### 354 **Setup of growth experiment with different initial cell numbers.**

355 HL60 cells were maintained in IMDM wGln, 20% FBS(heat inactivated), 1% P/S at a cell density  
356 between  $3 \times 10^5$  and  $2.5 \times 10^6$  cells/ml (GIBCO). Cells were always handled and maintained under  
357 sterile conditions (tissue culture hood; 37°C, 5% CO<sub>2</sub>, humidified incubator). At the beginning of the  
358 experiment, cells were collected, washed two times in PBS, and stained for vitality (Trypan blue  
359 GIBCO). The population of cells was first gated for morphology and then for vitality staining. Only  
360 Trypan negative cells were sorted (BD FACSAria II). The cells were sorted in a 384 well plate with  
361 IMDM wGln, 20% FBS(heat inactivated), and 1% P/S (GIBCO).

362 Cell population growth was monitored using a Leica microscope (heated environmental chamber  
363 and CO<sub>2</sub> levels control) with a motorized tray. Starting from Day 4, the 384 well plate was placed  
364 inside the environmental chamber every 24 hours. The images were acquired in a 3 × 3 grid for each  
365 well; after acquisition, the 9 fields were stitched into a single image. Software ImageJ was applied to  
366 identify and estimate the area occupied by "entities" in each image. The area (proportional to cell  
367 number) was used to follow the cell growth.

### 368 **Setup of reseeding experiment for growth pattern inheritance.**

369 HL60 cells were cultivated for 3 weeks, and then we chose one full well and one half full well. We  
370 supposed the full well was dominated by fast type cells, and the half-full well was dominated by  
371 moderate type cells, which had lower growth rates. We reseeded cells from these two wells and  
372 cultivated them in two 96-well (rows A-H, columns 1-12) plates. In each plate, B2-B11, D2-D11,  
373 and F2-F11 wells started with 78 fast cells, while C2-C11, E2-E11, and G2-G11 wells started with  
374 78 moderate cells. Rows A, H, columns 1, 12 had no cells and no media, and we found that wells  
375 in rows B, G, columns 2, 11, which were the outmost non-empty wells, evaporated much faster  
376 than inner wells. Therefore, the growth of cells in those wells was much slower than inner wells.  
377 Hence we only considered inner wells, where D3-D10 and F3-F10 started with fast cells, C3-C10 and  
378 E3-E10 started with moderate cells, namely 32 fast wells and 32 moderate wells in total. During the  
379 experiment, no media was added. Each day, we observed those wells to check whether their areas  
380 exceeded one-half of the whole well. The experiment was terminated after 20 days.

### 381 **Weighted Welch's $t$ -test.**

The weighted Welch's  $t$ -test is used to test the hypothesis that two populations have equal mean,  
while sample values have different weights (*Goldberg et al., 2005*). Assume for group  $i$  ( $i = 1, 2$ ), the  
sample size is  $N_i$  and the  $j$ th sample is the average of  $c_i^j$  independent and identically distributed

variables. Let  $X_i^j$  be the observed average for the  $j$ th sample. Set  $v_1 = N_1 - 1$ ,  $v_2 = N_2 - 1$ . Define

$$\begin{aligned}\bar{X}_i^W &= \left( \sum_{j=1}^{N_i} X_i^j c_j \right) / \left( \sum_{j=1}^{N_i} c_j \right), \\ s_{i,W}^2 &= \frac{N_i \left[ \sum_{j=1}^{N_i} (X_i^j)^2 c_j \right] / \left( \sum_{j=1}^{N_i} c_j \right) - N_i (\bar{X}_i^W)^2}{N_i - 1}, \\ t &= \frac{\bar{X}_1^W - \bar{X}_2^W}{\sqrt{\frac{s_{1,W}^2}{N_1} + \frac{s_{2,W}^2}{N_2}}}, \\ v &= \frac{\left( \frac{s_{1,W}^2}{N_1} + \frac{s_{2,W}^2}{N_2} \right)^2}{\frac{s_{1,W}^4}{N_1^2 v_1} + \frac{s_{2,W}^4}{N_2^2 v_2}}.\end{aligned}$$

382 If two populations have equal mean, then  $t$  satisfies the  $t$ -distribution with degree of freedom  $v$ .

383 The weighted Welch's  $t$ -test was applied to the growth experiment with different initial cell  
384 numbers, in order to determine whether the growth rates during exponential phase (5–50 area  
385 units) were different between groups. Here  $X_i^j$  corresponded to growth rate, and  $c_i^j$  corresponded  
386 to cell area. The  $p$ -value for  $N_0 = 10$ -cell group vs.  $N_0 = 4$ -cell group was  $2.12 \times 10^{-8}$ ; the  $p$ -value for  
387  $N_0 = 10$ -cell group vs.  $N_0 = 1$ -cell group was smaller than  $10^{-12}$ ; the  $p$ -value for  $N_0 = 4$ -cell group vs.  
388  $N_0 = 1$ -cell group was  $5.35 \times 10^{-5}$ . Therefore, the growth rate difference between any two groups  
389 was statistically significant.

### 390 Permutation Test.

391 The permutation test is a non-parametric method to test whether two samples are significantly  
392 different with respect to a statistic (e.g., sample mean) (*Hastie et al., 2016*). It is easy to calculate  
393 and fits our situation, thus we adopt this test rather than other more complicated tests, such as the  
394 Mann-Whitney test. For two samples  $\{x_1, \dots, x_m\}$ ,  $\{y_1, \dots, y_n\}$ , consider the null hypothesis: the mean  
395 of  $x$  and  $y$  are the same. For these samples, calculate the mean of the first sample:  $\mu_0 = \frac{1}{m} \sum x_i$ . Then  
396 we randomly divide these  $m + n$  samples into two groups with size  $m$  and  $n$ :  $\{x'_1, \dots, x'_m\}$ ,  $\{y'_1, \dots, y'_n\}$ ,  
397 such that each permutation has equal probability. For these new samples, calculate the mean of  
398 the first sample:  $\mu'_0 = \frac{1}{m} \sum x'_i$ . Then the two-sided  $p$ -value is defined as

$$p = 2 \min\{\mathbb{P}(\mu_0 \leq \mu'_0), 1 - \mathbb{P}(\mu_0 \leq \mu'_0)\}.$$

399 If  $\mu_0$  is an extreme value in the distribution of  $\mu'_0$ , then the two sample means are different.

400 In the reseeded experiment, the mean time of exceeding half well for the fast group was 11.4375  
401 days. For all  $\binom{64}{32}$  possible result combinations, only 7 combinations had equal or less mean time.  
402 Thus the  $p$ -value was  $2 \times 7 / \binom{64}{32} = 7.6 \times 10^{-18}$ . This indicated that the growth rate difference between  
403 fast group and moderate group was significant.

### 404 Model Details.

405 The simulation time interval was half day, but we only utilized the results in full days. For each initial  
406 cell, the probabilities of being fast, moderate or slow type,  $p_F, p_M, p_S$ , were 0.4, 0.4, 0.2.

407 Each half day, a fast type cell had probability  $d$  to die, and probability  $g_F$  to divide. The division  
408 produced two fast cells, capturing the intrinsic growth behavior that is to some extent inheritable.  
409 Denote the total cell number of previous day as  $N$ , then

$$g_F = g_0(1 - N^2/C^2) + \delta,$$

410 where  $\delta$  is a random variable that satisfies the uniform distribution on  $[-r, r]$ , and it is a constant for  
411 all cells in the same well. If  $g_F < 0$ , set  $g_F = 0$ . If  $g_F > 1 - d$ , set  $g_F = 1 - d$ .

412 In the simulation displayed, death rate  $d = 0.01$ , carrying capacity  $C = 40000$ , growth factor  
413  $g_0 = 0.5$ , and the range of random modifier  $r = 0.1$ .

414 Each half day, a moderate type cell had probability  $d$  to die, and probability  $g_M$  to divide. The  
415 division produced two moderate cells.  $g_M = g_F/1.5$ .

416 Similarly, each half day, a slow type cell had probability  $d$  to die, and probability  $g_S$  to divide. The  
417 division produced two slow-growing cells.  $g_S = g_F/3$ .

### 418 **Parameter scan.**

419 Since growth is measured by the area covered by cells, we could not experimentally verify most  
420 assumptions of our model, or determine the values of parameters. Therefore, we performed  
421 a parameter scan by evaluating the performance of our model for different sets of parameters.  
422 We adjusted 6 parameters: initial type probabilities  $p_F, p_M, p_S$ , death rate  $d$ , growth factor  $g_0$ , and  
423 random modifier  $r$ . We checked whether these 4 features observable in the experiment could be  
424 reproduced: growth of all wells in the  $N_0 = 10$ -cell group to saturation; existence of late-growing  
425 wells in the  $N_0 = 1$ -cell group; existence of non-growing wells in the  $N_0 = 1$ -cell group; difference in  
426 growth rates in the  $N_0 = 10$ -cell group and the  $N_0 = 1$ -cell group at the same population size. Table 3  
427 shows the results of the performance of simulations with the various parameter sets. Within a wide  
428 range of parameters, our model is able to replicate the experimental results shown in Figs. 1–3,  
429 indicating that our model is robust under perturbations.

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### 434 **Data availability**

435 The experimental data, simulation data, and corresponding code files could be found at  
436 <https://github.com/YueWangMathbio/Leukemia>.

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